

List of possible papers to read

lec 2 : (Physicist papers : Hurewicz, Krug-Spohn)^{80s}

These are sort of "intuitive" and probably good to read to see how physicists make guesses.

Kingman 1968 (Subadditive ergodic theorem)

This is a challenging, fun and technical paper.

Only worth reading if you enjoy analysis.

lec 3

v Berg-Kesten (strict inequality for time constant)

This is a lovely paper, and it talks about the types of weights you see on geodesics.

It has two IMPORTANT arguments. The Peierls argument and a "resampling"

argument. It's CHALLENGING.

Cox-Kesten (continuity of time constant) 1981. ^{Also in Kesten's aspects.}

I haven't read it. $F_n \Rightarrow F \Rightarrow M_{F_n} \rightarrow M_F$.

Burton-Keane (The ω cluster whenever it exists is unique. Density and uniqueness in percolation) Again very short paper, and was an idea called "finite energy" that ^{has been} ^{very} influential

Kesten (Aspects, $F(0) < p_c \Rightarrow M(e_1) > 0$)

lovely short argument, was the "vBk" inequality. Only need Prop 5.2.

Kesten ($p_c = \frac{1}{2}$ in Bernoulli bond percolation)

Challenging and fun. Uses duality of bond percolation. Great abstract. I haven't read it. Someone "explained" it to me, and I have long since forgotten.

lec 4a

- 1) Eden 1961 we will need in class. But there are a couple of ^{bounds} combinatorial χ_n that are quite interesting.
- 2) Turing 1952. This is a lovely ^{1D} χ_n model. But he did not extend this to 2D sheets.
- 3) Wieman and Reh 1978. On conjectures in first passage percolation theory. Ann. Prob. This shows that
$$\frac{T(0, L_n)}{n} \rightarrow \mu = \lim_{n \rightarrow \infty} \frac{T(0, n e_1)}{n}$$
 where L_n is the vertical line through $n e_1$.

Simulation : Fix a box of size $[-N, N]^2$.

and pick maybe n ^{iid} unif $[1, 2]$ weights. Repeat these weights by repeating the box $[-N, N]^2$ several times.

Then find $\frac{T(0, nx)}{n}$ for large n and many different values of x . This should give you the limit shape for periodic weights. Is it a polygon?

This is a stepping stone for a research project.

You can build on my python code at github.com (username arjunkc) if you like.

6.25

Lox-Durrett's limit shape theorem. There are two missing ingredients:

1) The lemma that says that $\exists K$ st

$$P\left(\sup_x \frac{T(0,x)}{|x|} < K\right) > 0$$

$$\text{is } \mathbb{E}[\min_i \tau_i^d] < \infty.$$

I think it's on pg 17 of their paper.

2) Why is $\mathbb{E}[\min_i \tau_i^d] < \infty$ an only if condition in their paper?

Qe 5

(1995 annals)

Haagstrom and Meester: Build a stationary ergodic system that can produce any ^{convex symmetric} limit shape

Kesten (Aspects, 1986, Corollary 8.4): If F has density f
 $f(x) = a \left(1 + o\left(\frac{1}{(\log x)}\right) \right)$ as $x \downarrow 0$ and finite mean, $a > 0$

Then

$$C_1 \frac{\log d}{ad} \leq M(F, d, e_1) \leq 11 \frac{\log d}{ad}$$

But let $M^+(F, d) = M(F, d, \underbrace{(e_1, e_1, \dots, e_1)}_{\substack{\text{diagonal} \\ \sqrt{d}}})$

and it's much faster here.

$$\frac{1}{6ca} \leq \underline{\lim} \sqrt{d}^+ \mu^+ \leq \overline{\lim} \sqrt{d}^+ \mu^+ \leq \frac{1}{2a}$$

So the shape cannot be a ball.

1988, Phys Rev. Letters

D-Dhar (Exponential FPP in high dimensions)

Angewandter - Tang (More FPP in higher dimensions)

lec 7 and 8

1981

Durrett and Liggett (Flat spots theorem)

We covered everything in Durrett and Liggett except for a couple of bounds due to Harris, and the limit shape theorem for the branching random walk.

Morhard (2002, strict inequalities): Several improvements to Durrett and Liggett. Also $M(\beta, \epsilon) < 1$ where β is defined in the notes.

Damron-Auffinger (2012, Differentiability): Showed that at the edge of the percolation cone, the shape is differentiable.

Durrett's: Annals of probability paper on Directed Percolation. 1984. This is a long and detailed paper. Very well written and gives a broad overview of the subject.

lec 07: Durrett's 1980 paper on the contact process. (on the 1D Contact Process)

Harris 1978 red valued Markov Process (AOP)

Lemma 6.1 of Durrett 1980. Helps show

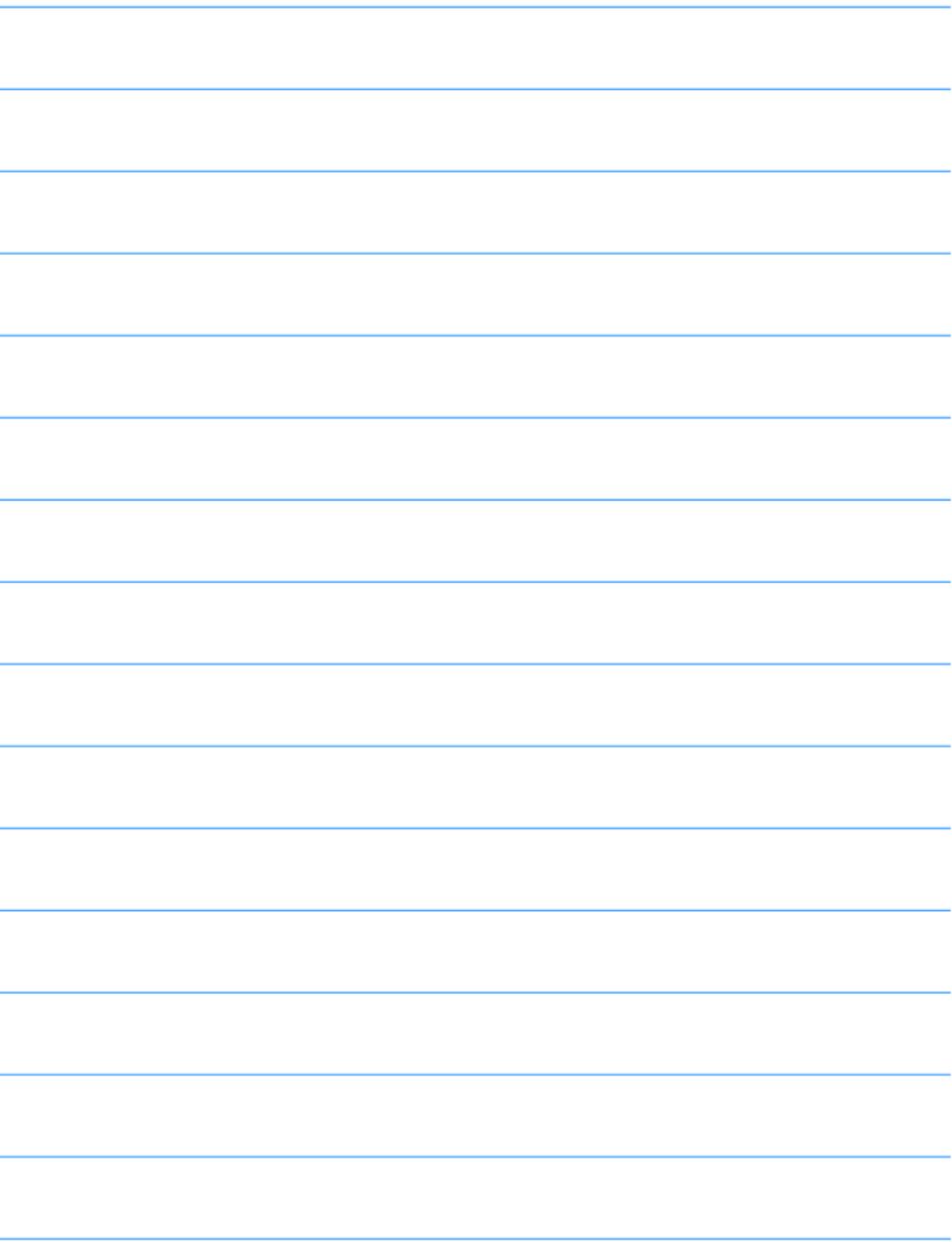
$$E [r_n^{BU\{0\}} - r_n^B] \geq 1$$

Lec 9

Cramer's theorem: Any book on large deviation theory.

Dembo / Zeitouni, Klenke (Prob. Theory), Ellis (Entropy
Large deviations and Statistical mechanics)

Newman - Piza (1995) Annals of Prob. Loebl, classic paper. Lots of information about fluctuation shape, etc.



lect 10, 11, 12

Many inequalities in this lecture.

- 1) S. Chatterjee, "Super concentration and other topics"
- 2) Kahn-Kalai-Lindal. Boolean Influence Inequalities. To understand some of the "CS" aspects of these inequalities.
- 3) Talagrand 1994. Russo's approximate 0-1 law. On the logarithmic improvement to Efron-Stein.
- 4) Prove the Gaussian-Poincaré inequality. It's a 10 min, entertaining proof. You can find this in many places, including Chatterjee's book.
- 5) M. Ledoux. "Concentration of Measures." Another book to find some of these topics. You can probably find a proof of 4) here.

There are lots of lovely papers in this area.

The wax of the B&W is kind of amazing too.

lec 13 (separated occurrence inequalities)

- 1) The original BK paper 80s: 1985 Inequalities with application to reliability and percolation
- 2) Chayes-Chays and Landall. 1999. Reimer's proof of the BK inequality

lec 16 (Kesten's Lemma)

1) 1980 (On the time constant and path length of FPP)

Has a proof of $E[W] < \infty$

2) Grimmett Percolation Theorem 5.75 . Proves that

the expected size of the percolation cluster $< \infty$

for $p < p_c$

3) The text book also suggests Deminil-Lopin and

Tenera for this.