Math 210

Problem Set # 1

Question 1. A fair coin is flipped 7 times. Let X = total number of heads. The sample space consists of all possible sequences of outcomes of the 7 flips.

a) Write down three possible outcomes ω from the sample space Ω.

b) How many outcomes are in the sample space?

c) Compute $P(\{\omega\})$ for each outcome ω you wrote down in part (a).

d) Compute $X(\omega)$ for each outcome ω you wrote down in part (a).

e) Find $P(X \leq 5)$. Hint: Find $P(X > 5)$ first.

Question 2. Consider a coin where the probability of heads is $0 < p < 1$. Do **not** assume $p = 1/2$. Flip it until the first tails occurs. Let X = number of flips needed to see the first tails.

1. How many possible outcomes are in the sample space?
2. Find $P(X = 1)$, $P(X = 2)$, and $P(X = 3)$.
3. Write down a formula for $P(X = k)$, where k is a positive integer.

Question 3. In E.O. Thorp's *The Mathematics of Gambling* he considers a strategy for blackjack due to Baldwin et al in which the probability that the player wins (and the house dealer loses) is .5005. Suppose this is accurate. If you have \$100 to gamble and on one turn you gamble $100y$ dollars where y is a number in $[0, 1]$ then after one turn your resulting fortune is

$$\begin{cases} 100(1+y) & \text{with probability } 0.5005 \\ 100(1-y) & \text{with probability } 0.4995. \end{cases}$$

Let X be the random variable representing your fortune after one turn of the game.

1. Find the expectation $E[X]$ as a function of y.
2. Find the value of y in $[0, 1]$ which maximizes $E[X]$.
3. Consider the random variable
   $$k = \log(X/100)$$
   known as the *logarithmic return rate*. Calculate $E[k]$ as a function of y.
4. Find the value of y in $[0, 1]$ which maximizes $E[k]$.