

**1.1** The Smiths receive the paper every morning and place it on a pile after reading it. Each afternoon, with probability  $1/3$ , someone takes all the papers in the pile and puts them in the recycling bin. Also, if ever there are at least five papers in the pile, Mr. Smith (with probability 1) takes the papers to the bin. Consider the number of papers in the pile in the evening. Is it reasonable to model this by a Markov chain? If so, what are the state space and transition matrix?

**1.2** Consider a Markov chain with state space  $\{0, 1\}$  and transition matrix

$$\mathbf{P} = \begin{matrix} & \begin{matrix} 0 & 1 \end{matrix} \\ \begin{matrix} 0 \\ 1 \end{matrix} & \begin{bmatrix} 1/3 & 2/3 \\ 3/4 & 1/4 \end{bmatrix} \end{matrix}.$$

Assuming that the chain starts in state 0 at time  $n = 0$ , what is the probability that it is in state 1 at time  $n = 3$ ?

**1.3** Consider a Markov chain with state space  $\{1, 2, 3\}$  and transition matrix

$$\mathbf{P} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} .4 & .2 & .4 \\ .6 & 0 & .4 \\ .2 & .5 & .3 \end{bmatrix} \end{matrix}.$$

What is the probability in the long run that the chain is in state 1? Solve this problem two different ways: 1) by raising the matrix to a high power; and 2) by directly computing the invariant probability vector as a left eigenvector.

**1.4** Do the same for the transition matrix

$$\mathbf{P} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} .2 & .4 & .4 \\ .1 & .5 & .4 \\ .6 & .3 & .1 \end{bmatrix} \end{matrix}.$$