Quiz 1

Thirteen cards are numbered $1, \ldots, 13$, shuffled and dealt one at a time. Say a match occurs on deal k if the k^{th} card revealed is number k.

Let N be the total # of matches that occur in the thirteen cards. Determine $E[N]$

Hint: $N = 1_{A_1} + \cdots + 1_{A_{13}}$ where A_k is the event that a match occurs on deal k.

Solution: $A_i = \{\text{card } i \text{ is dealt on deal } i\}$

$P(A_1) = \frac{1}{13}$ clearly

let E_2 be the event that 2 already occurred in deal 1.

$$P(A_2) = \cancel{P(A_2 | E_2)}^{0} P(E_2) + P(A_2|E_2^c)P(E_2^c)$$

$$= \frac{1}{12} \cdot \frac{12}{13} = \frac{1}{13}$$

One can generalize this idea to show $P(A_n) = \frac{1}{13}$ (You don't need a full proof on the quiz)

Another way: $\_\ \_\ \_\ \_\ \_\ \_\ \underline{k}\ \_\ \_\ \_$

$$P(A_k) = P(k \text{ occurs on the } k^{th} \text{ draw}) = \frac{(13-1)!}{13!} = \frac{1}{13}$$

$$E[N] = E[1_{A_1} + \cdots + 1_{A_{13}}]$$

$$= E[1_{A_1}] + \cdots + E[1_{A_{13}}] = P(A_1) + \cdots + P(A_{13})$$

$$= 13\, P(A_1) = 1 \ !$$